Week 7 Discussion

CAS CS 132: Geometric Algorithms

October 16, 2023

During discussion sections, we will go over three problems.

- The first will be a warm-up question, to help you verify your understanding of the material.
- The second will be a solution to a problem on the assignment of the previous week.
- The third will be a problem similar to one on the assignment of the following week.

The remainder of the time will be dedicated to open Q&A.

1 Matrix Inverses (Warm Up)

For each matrix A, determine if it is invertible. If it is, compute its inverse A^{-1} and demonstrate that it is an inverse by computing $A^{-1}A$ and AA^{-1}.

A.

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 0 & -1 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

B.

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix}$$

Solution.

2 Span and Linear Independence (Midterm)

Consider the following vectors in $\mathbb{R}^4$.

$$\mathbf{v}_1 = \begin{bmatrix} -3 \\ 4 \\ 3 \\ 7 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 2 \end{bmatrix} \quad \mathbf{v}_3 = \begin{bmatrix} 0 \\ -2 \\ -1 \\ -1 \end{bmatrix} \quad \mathbf{v}_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -2 \end{bmatrix}$$

A. Determine if $\mathbf{v}_1$ is in $\mathsf{span}\{\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$. Justify your answer. In particular, if $\mathbf{v}_1$ is in $\mathsf{span}\{\mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$, then write $\mathbf{v}_1$ as a linear combination of $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$.

B. Determine if the vectors $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$ are linearly independent. Justify your answer. In particular, if they are linearly dependent, then write a dependence relation for them (that is, write the zero vector $\mathbf{0}$ as a linear combination of the vectors $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$).

C. Determine if the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are linearly independent. Justify your answer. In particular, if they are linearly dependent, then write a dependence relation for them.

Solution.

3 3D Rotation Matrices

The matrices used for rotating counter-clockwise by θ around the x-, y-, and z-axes are

$$R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \quad R_{y,\theta} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \quad R_{z,\theta} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Note that difference in sign for $R_{y,\theta}$. This is due to a convention called the *right-hand rule* which says that the orientation of the positive directions of each axis is given by thinking of the index finger, middle finger, and thumb of our right hand as the x-, y-, and $z-$axes (and that when we talk about counter-clockwise around an axis, we mean counter-clockwise when the positive axis is pointing towards us).

A. Compute $\mathbf{v} = R_{z,45^\circ}[1\ (-1)\ \sqrt{2}]^T$. Recall that $\cos 45^\circ = \sin 45^\circ = \frac{\sqrt{2}}{2}$.

B. Computer $R_{y,45^\circ}\mathbf{v}$, where $\mathbf{v}$ is the vector from the previous part.

Try to draw, to the best of your ability, what is happening to this vector after each transformation.

Solution.