Week 10 Discussion

CAS CS 132: Geometric Algorithms

November 6, 2023

During discussion sections, we will go over three problems.

- The first will be a warm-up question, to help you verify your understanding of the material.
- The second will be a solution to a problem on the assignment of the previous week.
- The third will be a problem similar to one on the assignment of the following week.

The remainder of the time will be dedicated to open Q&A.

1 Column Space and Null Space (Warm Up)

Consider the following matrix

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & -1 \\ -3 & 1 & 5 \end{bmatrix}$$

A. Find the reduced echelon form of A.

B. Write down a general form solution for the equation $A\mathbf{x} = \mathbf{0}$.

C. Find a basis for $\operatorname{Col} A$.

D. Find a basis for $\operatorname{Nul} A$.

E. Find a linear equation whose solution set (i.e., those points in the plane represented by the linear equation) is $\operatorname{Col} A$.

Solution.

2 Order of Matrix Multiplcation

The matrices for the 2D transformations (on homogeneous coordinates) of translation and counterclockwise rotation about the origin are

$$T_{x,y} = \begin{bmatrix} 1 & 0 & x \\ 0 & 1 & y \\ 0 & 0 & 1 \end{bmatrix} \qquad R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A. Show that $T_{1,1}R_{\pi/4} \neq R_{\pi/4}T_{1,1}$. Recall that $\pi/4$ in radians is $45°$ and $\cos(\pi/4) = \sin(\pi/4) = \sqrt{2}/2$.

B. If we want to do rotation *and then* translation, which of the two matrices in the previous part do we want to use?

C. Draw the effects $T_{1,1}R_{\pi/4}$ and $R_{\pi/4}T_{1,1}$ on the unit square on two separate graphs.

Solution.

3 Intersections and Unions of Subspaces

Let H_1 and H_2 be arbitrary subspaces of $\mathbb{R}^n$.

A. The *intersection* of H_1 and H_2, written $H_1 \cap H_2$, is the set of vectors which appear in *both* H_1 and H_2:

$$H_1 \cap H_2 = \{\mathbf{v} : \mathbf{v} \in H_1 \text{ and } \mathbf{v} \in H_2\}$$

Show that $H_1 \cap H_2$ is a subspace. (In other words, the intersection of two spans is a span).

B. The *union* of H_1 and H_2, written $H_1 \cup H_2$, is the set of vectors which appear in *at least one of* H_1 and H_2:

$$H_1 \cup H_2 = \{\mathbf{v} : \mathbf{v} \in H_1 \text{ or } \mathbf{v} \in H_2\}$$

Give an explicit example of two subspaces $\mathbb{R}^2$ whose union is *not* a subspace of $\mathbb{R}^2$. *Hint.* Pick pretty much any two vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^2$ and take the two subspaces to be $\text{span}\{\mathbf{u}\}$ and $\text{span}\{\mathbf{v}\}$.

Solution.