

# Week 11 Discussion

CAS CS 132: Geometric Algorithms

November 13, 2023

During discussion sections, we will go over three problems.

- The first will be a warm-up question, to help you verify your understanding of the material.
- The second will be a solution to a problem on the assignment of the previous week.
- The third will be a problem similar to one on the assignment of the following week.

The remainder of the time will be dedicated to open Q&A.

# 1 Eigenvalues, Eigenvectors, Eigenspaces

- A. Find an invertible  $2 \times 2$  matrix with no eigenvalues.
- B. Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be the linear transformation which projects points onto the  $x_1x_2$ -plane. Find the eigenvalues and bases for the corresponding eigenspaces of the matrix implementing this transformation *without doing any calculations*. Then write down the matrix implementing this transformation and find its characteristic polynomial. Check that the eigenvalues you get from the characteristic polynomial are the same.
- C. Find the eigenvalues and bases for the corresponding eigenspace of

$$\begin{bmatrix} 1 & -4 \\ -3 & 5 \end{bmatrix}$$

*Solution.*

## 2 Complement of the Column Space

Let  $A$  be a  $5 \times n$  matrix such that  $\text{rank } A = 4$ , which has an LU decomposition where

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 4 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 \\ 0 & 3 & -3 & 0 & 1 \end{bmatrix}$$

Determine if  $\mathbf{v}$  in  $\text{Col } A$ , where

$$\mathbf{v} = \begin{bmatrix} 2 \\ -5 \\ -11 \\ 5 \\ -12 \end{bmatrix}$$

*Solution.*

### 3 Characteristic Polynomials

Find the characteristic polynomial for the matrix

$$A = \begin{bmatrix} 1 & -1 & 5 \\ 0 & 2 & 4 \\ 0 & 1 & 5 \end{bmatrix}$$

Use this to determine the eigenvalues of  $A$ .

*Solution.*