Week 13 Discussion

CAS CS 132: Geometric Algorithms

November 27, 2023

During discussion sections, we will go over three problems.

- The first will be a warm-up question, to help you verify your understanding of the material.
- The second will be a solution to a problem on the assignment of the previous week. **But not this week, there was no assignment last week.**
- The third will be a problem similar to one on the assignment of the following week.

The remainder of the time will be dedicated to open Q&A.

1 Inner Products, Norms, Orthogonality

The first two problems come from *Linear Algebra and its Applications.*

A. Given vectors

$$\mathbf{u} = \begin{bmatrix} 2 \\ -5 \\ -1 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} -7 \\ -4 \\ 6 \end{bmatrix}$$

compute (by hand) $\mathbf{u} \cdot \mathbf{v}$, $\|\mathbf{u}\|^2$, $\|\mathbf{v}\|^2$, and $\|\mathbf{v} + \mathbf{u}\|^2$

B. Show that if $\mathbf{y}$ is orthogonal to $\mathbf{u}$ and $\mathbf{v}$, then it is orthogonal to $\mathbf{u} + \mathbf{v}$.

C. Find a linearly independent set of three nonzero vectors in $\mathbb{R}^4$, all of which are orthogonal to

$$\begin{bmatrix} 1 \\ 3 \\ -1 \\ 3 \end{bmatrix}$$

Solution.

2 Damping without a Matrix

Suppose that A is a $n \times n$ stochastic matrix. According to what we talked about in lecture, the matrix we use to perform PageRank is

$$(1-\alpha)A + \frac{\alpha \mathbf{1}^{n\times n}}{n}$$

where $\mathbf{1}^{n\times n}$ here is the $n \times n$ all-ones matrix. However, for the assignment this week, you will not be able to build this matrix because it is too dense. Given a vector $\mathbf{v}$ from $\mathbb{R}^n$, write down an expression for

$$\left((1-\alpha)A + \frac{\alpha \mathbf{1}^{n\times n}}{n}\right)\mathbf{v}$$

which does not require building the matrix $\mathbf{1}^{n\times n}$. *Hint.* The expression should be of the form

$$(1-\alpha)A\mathbf{v} + \mathbf{u}$$

where $\mathbf{u}$ is a vector depending on $\mathbf{v}$ and the all-ones *vector* $\mathbf{1}^n \in \mathbb{R}^n$.

Solution.