Week 15 Discussion

CAS CS 132: Geometric Algorithms

December 11, 2023

During discussion sections, we will go over three problems.

- The first will be a warm-up question, to help you verify your understanding of the material.
- The second will be a solution to a problem on the assignment of the previous week.
- The third will be a problem similar to one on the assignment of the following week.

The remainder of the time will be dedicated to open Q&A.

1 LAA 6.6.10 (Warm Up)

Suppose radioactive substances A and B have decay constants of 0.02 and 0.07, respectively. If a mixture of these two substances at time $t = 0$ contains M_A grams of A and M_B grams of B, then a model for the total amount of y of the mixture present at time t is

$$y = M_A e^{-0.2t} + M_B e^{-0.7t}$$

Suppose the initial amounts M_A and M_B are unknown, but a scientist is able to measure the total amounts present at several times and records the following points (t_i, y_i):

$$(10, 21.34), (11, 20.68), (12, 20.05), (14, 18.87), (15, 18.30)$$

Find the design matrix X such that the least squares solution of

$$X \begin{bmatrix} M_A \\ M_B \end{bmatrix} = \begin{bmatrix} 18.30 \\ 18.87 \\ 20.05 \\ 20.68 \\ 21.34 \end{bmatrix}$$

provides an estimate for M_A and M_B.

Solution.

2 Least Square (Homework 11)

Let C be an arbitrary $m \times n$ matrix of rank k and let $\mathbf{b}$ be an arbitrary vector in $\mathbb{R}^m$. Find an expression for the maximum size of a linearly independent set of least squares solutions for $C\mathbf{x} = \mathbf{b}$ where $\mathbf{b} \neq 0$.

Solution.

3 Design Matrices

Write Python code (or pseudocode) which builds the design matrix for modeling with the function

$$f(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \cdots + \beta_k x^k$$

given a feature vector $\mathbf{x}$ (i.e., a vector representing the independent variable for a collection of data points) represented as a 1D NumPy array, and an integer k which determines the degree used in f.

Solution.