Homework 1 Solutions

CAS CS 132

Fall 2024

Problem 1.1

$$\begin{bmatrix} 2 & 4 & 29 \\ -6 & -10 & -75 \end{bmatrix} \xrightarrow{R_2 \leftarrow R_2 + 3R_1} \begin{matrix} \\ +6 \quad +12 \quad +87 \end{matrix}$$

$$\begin{bmatrix} 2 & 4 & 29 \\ 0 & 2 & 12 \end{bmatrix} \xrightarrow{R_2 \leftarrow R_2/2}$$

$$\begin{bmatrix} 2 & 4 & 29 \\ 0 & 1 & 6 \end{bmatrix} \begin{matrix} \\ -0 \quad -4 \quad -24 \end{matrix} \xrightarrow{R_1 \leftarrow R_1 - 4R_2}$$

$$\begin{bmatrix} 2 & 0 & 5 \\ 0 & 1 & 6 \end{bmatrix} \xrightarrow{R_1 \leftarrow R_1/2}$$

$$\begin{bmatrix} 1 & 0 & 5/2 \\ 0 & 1 & 6 \end{bmatrix}$$

$x = 5/2$

$y = 6$

Problem 1.2

$$\begin{bmatrix} 1 & 3 & -3 & -2 \\ 2 & 7 & 7 & -5 \\ 1 & -2 & -1 & 3 \end{bmatrix} \quad R_2 \leftarrow R_2 - 2R_1 \longrightarrow$$

$$\begin{bmatrix} 1 & 3 & -3 & -2 \\ 0 & 1 & 13 & -1 \\ 1 & -2 & -1 & 3 \end{bmatrix} \quad R_3 \leftarrow R_3 - R_1 \longrightarrow$$

$$\begin{bmatrix} 1 & 3 & -3 & -2 \\ 0 & 1 & 13 & -1 \\ 0 & -5 & 2 & 5 \end{bmatrix} \quad R_3 \leftarrow R_3 + 5R_2 \longrightarrow$$

$$\begin{bmatrix} 1 & 3 & -3 & -2 \\ 0 & 1 & 13 & -1 \\ 0 & 0 & 67 & 0 \end{bmatrix} \quad R_3 \leftarrow R_3 / 67 \longrightarrow$$

$$\begin{bmatrix} 1 & 3 & -3 & -2 \\ 0 & 1 & 13 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad R_2 \leftarrow R_2 - 13R_3 \longrightarrow$$

$$\begin{bmatrix} 1 & 3 & -3 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad R_1 \leftarrow R_1 + 3R_3 \longrightarrow$$

$$\begin{bmatrix} 1 & 3 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \xrightarrow{R_1 \leftarrow R_1 - 3R_2}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$x = 1$

$y = -1$

$z = 0$

Problem 2

$$\begin{bmatrix} 1 & 2 & -1 & 5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 5 \\ 2 & 2 & 1 & 1 \end{bmatrix} \quad R_2 \leftrightarrow R_4 \longrightarrow$$

$$\begin{bmatrix} 1 & 2 & -1 & 5 \\ 2 & 2 & 1 & 1 \\ 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 1 \end{bmatrix} \quad R_1 \leftarrow R_1 + R_2 \longrightarrow$$

$$\begin{bmatrix} 3 & 4 & 0 & 6 \\ 2 & 2 & 1 & 1 \\ 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 1 \end{bmatrix} \quad R_3 \leftarrow R_3 - R_4 \longrightarrow$$

$$\begin{bmatrix} 3 & 4 & 0 & 6 \\ 2 & 2 & 1 & 1 \\ 0 & -1 & 0 & 4 \\ 0 & 1 & 0 & 1 \end{bmatrix} \quad R_2 \leftarrow 2R_2 \longrightarrow \quad \begin{bmatrix} 3 & 4 & 0 & 6 \\ 4 & 4 & 2 & 2 \\ 0 & -1 & 0 & 4 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Problem 3.1

augmented: $\begin{bmatrix} 1 & -2 & 5 & 0 & 6 \\ -2 & 6 & -11 & 7 & -8 \\ 5 & -10 & 25 & 3 & 30 \end{bmatrix}$

rref: $\begin{bmatrix} 1 & 0 & 4 & 0 & 10 \\ 0 & 1 & -1/2 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

infinitely many solutions

Problem 3.2

augmented: $\begin{bmatrix} 1 & -3 & 4 & 0 \\ -1 & 6 & 1 & 4 \\ 0 & 23 & 5 & 9 \end{bmatrix}$

rref: $\begin{bmatrix} 1 & 0 & 0 & -37/20 \\ 0 & 1 & 0 & 1/4 \\ 0 & 0 & 1 & 13/20 \end{bmatrix}$

unique solution

Problem 3.3

augmented: $\begin{bmatrix} 1 & 5 & 3 & -4 \\ 1 & 6 & 7 & -13 \\ -2 & -12 & -14 & 25 \end{bmatrix}$

rref: $\begin{bmatrix} 1 & 0 & -17 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

no solution

Problem 4

Company C

Explanation: The data as given is incorrect by assumption. There are 5 possibilities for the correct data, gotten by subtracting 13 from one of the positions in the last column. Solving the system give by these 5 possibilities yields 5 possible costs-per-unit of each part. Only one solution has all positive values, the case of subtracting 13 from the total of company C.