

Homework 2 Solutions

CAS CS 132

Fall 2024

Problem 1

①

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

②

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

③

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

④

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

⑤

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

⑥

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

⑦

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

⑧

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Problem 2.1

$$\left[\begin{array}{cccc} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

no solutions

Problem 2.2

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 1 \end{array} \right]$$

$$x_1 = -3$$

$$x_2 = 3$$

$$x_3 = -6$$

$$x_4 = 1$$

Problem 2.3

$$\left[\begin{array}{ccccccc} 0 & 1 & 0 & 2 & -1 & 0 & 0 & 30 \\ 0 & 0 & 1 & -1 & 4 & 0 & 0 & -14 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 12 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

x_1 is free

$$x_2 = 30 - 2x_4 + x_5$$

$$x_3 = (-14) + x_4 - 4x_5$$

x_4 is free

x_5 is free

$$x_6 = 6$$

$$x_7 = 12$$

Problem 2.4

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

no solution

Problem 3

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{3} & 1 \\ 0 & 1 & \frac{1}{4} & 4 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$x_1 = 1 + \frac{1}{3}x_3$$

$$x_2 = 4 - \frac{1}{4}x_3$$

x_3 is free

$$(5, 1, 12)$$

Problem 4.1

$$x_1 = 4 + 3x_2 - 2x_4$$

x_2 is free

$$x_3 = (-2) + x_4$$

x_4 is free

Problem 4.2

$$\left[\begin{array}{ccccc} 1 & -3 & 0 & 2 & 4 \\ 0 & 0 & 1 & -1 & -2 \end{array} \right]$$

Note: The solution can have any number of all-zero rows at the end