Homework 3 Solutions

CAS CS 132

Fall 2024

Problem 1

$$\begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & -3 & 1 \end{bmatrix}$$

Explanation (Not Required): Two distinct planes intersect at a line so we know there is exactly one free variable. The RREF cannot have a pivot in the third column because z has different values in the two given solutions. So the RREF has form

$$\begin{bmatrix} 1 & 0 & a & c \\ 0 & 1 & b & d \end{bmatrix}$$

Solve for a, b, c and d.

Problem 2.1

$$\begin{bmatrix} 10 & -47 & 31 & -14 & 42 \\ -33 & -1 & -32 & 22 & -23 \\ -5 & -2 & 11 & 12 & 98 \\ -30 & -25 & 39 & 42 & 11 \\ -3 & 24 & 25 & 3 & -87 \end{bmatrix}$$

Not in the span

Problem 2.2

$$\begin{bmatrix} 10 & -47 & 31 & -14 & 87 \\ -33 & -1 & -32 & 22 & -19 \\ -5 & -2 & 11 & 12 & -24 \\ -30 & -25 & 39 & 42 & -61 \\ -3 & 24 & 25 & 3 & -79 \end{bmatrix}$$

$$\vec{v} = \vec{v}_1 - 2\vec{v}_2 - \vec{v}_3 - \vec{v}_4$$

Problem 3.1

$$\begin{array}{c} \begin{bmatrix} 1 & 1 & b_1 \\ 0 & 1 & b_2 \\ -2 & 1 & b_3 \end{bmatrix} \\ \begin{matrix} +2 & +2 & +2b_1 \end{matrix} \end{array} \xrightarrow{R_3 \leftarrow R_3 + 2R_1}$$

$$\begin{array}{c} \begin{bmatrix} 1 & 1 & b_1 \\ 0 & 1 & b_2 \\ 0 & 3 & b_3 + 2b_1 \end{bmatrix} \\ \begin{matrix} -0 & -3 & -3b_2 \end{matrix} \end{array} \xrightarrow{R_3 \leftarrow R_3 - 3R_2}$$

$$\begin{bmatrix} 1 & 1 & b_1 \\ 0 & 1 & b_2 \\ 0 & 0 & b_3 + 2b_1 - 3b_2 \end{bmatrix}$$

$$2x_1 - 3x_2 + x_3 = 0$$

Problem 3.2

$$\begin{bmatrix} 1 & 0 & b_1 \\ 1 & 1 & b_2 \\ -5 & -1 & b_3 \end{bmatrix} \xrightarrow{R_2 \leftarrow R_2 - R_1}$$

$-1 \quad -0 \quad -b_1$

$$\begin{bmatrix} 1 & 0 & b_1 \\ 0 & 1 & b_2 - b_1 \\ -5 & -1 & b_3 \end{bmatrix} \xrightarrow{R_3 \leftarrow R_3 + 5R_1}$$

$+5 \quad +0 \quad +5b_1$

$$\begin{bmatrix} 1 & 0 & b_1 \\ 0 & 1 & b_2 - b_1 \\ 0 & -1 & b_3 + 5b_1 \end{bmatrix} \xrightarrow{R_3 \leftarrow R_3 + R_2}$$

$+1 \quad +b_2 - b_1$

$$\begin{bmatrix} 1 & 0 & b_1 \\ 0 & 1 & b_2 - b_1 \\ 0 & 0 & b_3 + 4b_1 + b_2 \end{bmatrix}$$

$4(2) + (-3) + 2 = 7$

$4x_1 + x_2 + x_3 = 7$

Problem 4

$x_1 = 12 - 2x_3 - 3x_5$

$x_2 = 1 - x_3 + 9x_5$

x_3 is free

$x_4 = x_5$

x_5 is free

$x_6 = -6$

Problem 5.1

$$\begin{bmatrix} 4 & 2 & 1 & 13 \\ 9 & 3 & 1 & 25 \\ 4 & -2 & 1 & 5 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$y = 2x^2 + 2x + 1$$

Problem 5.2

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{y-1}{x-1} \\ 0 & 0 & 1 & \frac{x-y}{x-1} \end{bmatrix}$$

Explanation: The points (x,y) $(1,1)$ and $\left(\frac{x+1}{2}, \frac{y+1}{2}\right)$ lie on a line with slope $\frac{y-1}{x-1}$ and y-intercept $\frac{x-y}{y-1}$. So the interpolation would give the solution

$$y = 0x^2 + \left(\frac{y-1}{x-1}\right)x + \frac{x-y}{x-1}$$

Note: $\frac{x-y}{x-1} = 1 - \frac{y-1}{x-1} = y - \frac{x(y-1)}{x-1}$