Homework 5 Solutions

CAS CS 132

Fall 2024

Problem 1.1

$$\begin{bmatrix} 2 & 3 & -1 \\ 7 & 4 & 3 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & -1 \\ 7 & 4 & 3 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2(4) + 3(2) \\ 7(4) + 4(2) \\ 0 + (-1)2 \end{bmatrix} = \begin{bmatrix} 14 \\ 36 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & -1 \\ 7 & 4 & 3 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 + 3 - 3 \\ 7 + 4 + 9 \\ 0 + -1 + 6 \end{bmatrix} = \begin{bmatrix} 2 \\ 20 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 14 & 2 \\ 7 & 36 & 20 \\ 0 & -2 & 5 \end{bmatrix}$$

Problem 1.2

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \\ 5 & 6 & 7 \end{bmatrix} =$$

$$\begin{bmatrix} (1+3+5) & (2+4+6) & (3+5+7) \\ (2+4) & (3+5) & (4+6) \\ (1+3+5) & (2+4+6) & (3+5+7) \end{bmatrix} =$$

$$\begin{bmatrix} 9 & 12 & 15 \\ 6 & 8 & 10 \\ 9 & 12 & 15 \end{bmatrix}$$

Problem 1.3

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{pmatrix} = 1+4+9+16+25+36$$

5 14 30 55 91

$$= 91$$

Problem 1.4

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 6 & 8 & 10 & 12 \\ 3 & 6 & 9 & 12 & 15 & 18 \\ 4 & 8 & 12 & 16 & 20 & 24 \\ 5 & 10 & 15 & 20 & 25 & 30 \\ 6 & 12 & 18 & 24 & 30 & 36 \end{bmatrix}$$

Note: Not much work needs to be shown

Problem 1.5

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \cdots \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}}_{10} = \begin{bmatrix} 2^9 & 2^9 \\ 2^9 & 2^9 \end{bmatrix}$$

$$= \begin{bmatrix} 512 & 512 \\ 512 & 512 \end{bmatrix}$$

Problem 2.1

neither

not one-to-one: $\exists\ \vec{a}_1 = \vec{a}_3$, columns are not LI

not onto: more rows than columns (never as many pivots as rows)

Problem 2.2

one-to-one, not onto

one-to-one: any set with a single nonzero vector is LI

not onto: more rows than columns

Problem 2.3

onto, not one-to-one

onto: $\sim \begin{bmatrix} 1 & 4 & 3 & 0 \\ 0 & -7 & -3 & 0 \\ 0 & 0 & 7 & 0 \end{bmatrix}$ pivot per row

not one-to-one: $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ column, columns

not LI

Problem 2.4

both

one pivot per column and per row

Problem 3.1

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \mapsto \begin{bmatrix} x_2 \\ x_1 \\ x_3 \\ x_4 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \mapsto \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \mapsto \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \mapsto \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \mapsto \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 3.2

$$\begin{bmatrix}1\\0\\0\\0\end{bmatrix} \mapsto \begin{bmatrix}0\\1\\0\\0\end{bmatrix} \qquad \begin{bmatrix}0\\1\\0\\0\end{bmatrix} \mapsto \begin{bmatrix}-2\\0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}0\\0\\1\\0\end{bmatrix} \mapsto \begin{bmatrix}1\\1\\-3\\0\end{bmatrix} \qquad \begin{bmatrix}0\\0\\0\\1\end{bmatrix} \mapsto \begin{bmatrix}1\\0\\-3\\9\end{bmatrix}$$

$$\begin{bmatrix}0 & -2 & 1 & 1\\ 1 & 0 & 1 & 0\\ 0 & 0 & -3 & -3\\ 0 & 0 & 0 & 9\end{bmatrix}$$

domain : $\mathbb{R}^4$

codomain : $\mathbb{R}^4$

Problem 3.3

$$\begin{bmatrix} -7 & 9 & 1 \\ -3 & 4 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} -7 & 9 & 0 \\ -3 & 4 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 9 \\ 0 & 1 & 7 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = T\left(-4\begin{bmatrix} -7 \\ -3 \end{bmatrix} - 3\begin{bmatrix} 9 \\ 4 \end{bmatrix}\right) =$$

$$(-4)T\left(\begin{bmatrix} -7 \\ -3 \end{bmatrix}\right) - 3\,T\left(\begin{bmatrix} 9 \\ 4 \end{bmatrix}\right) = \begin{bmatrix} -23 \\ -12 \\ -16 \\ -24 \\ -13 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = T\left(9\begin{bmatrix} -7 \\ -3 \end{bmatrix} + 7\begin{bmatrix} 9 \\ 4 \end{bmatrix}\right) =$$

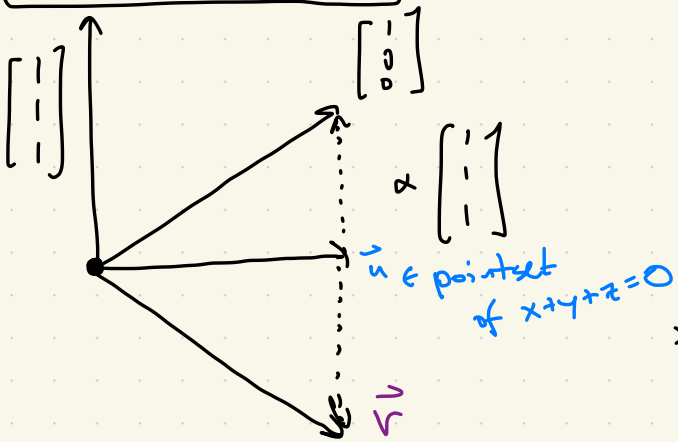
$$9\,T\left(\begin{bmatrix} -7 \\ -3 \end{bmatrix}\right) + 7\,T\left(\begin{bmatrix} 9 \\ 4 \end{bmatrix}\right) = \begin{bmatrix} 55 \\ 27 \\ 36 \\ 56 \\ 30 \end{bmatrix}$$

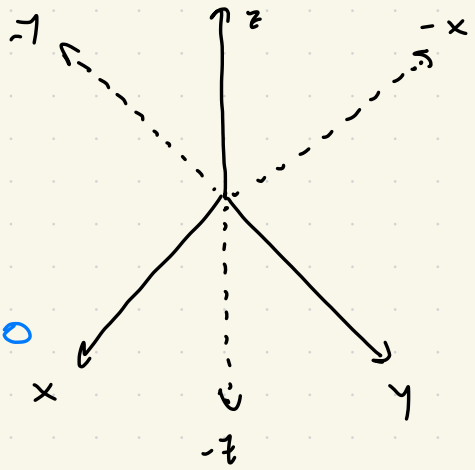
$$\begin{bmatrix} -23 & 55 \\ -12 & 27 \\ -16 & 36 \\ -24 & 56 \\ -13 & 30 \end{bmatrix}$$

domain: $\mathbb{R}^2$

codomain: $\mathbb{R}^5$

Problem 4.1





$$\begin{bmatrix}1\\0\\0\end{bmatrix} - \begin{bmatrix}\alpha\\\alpha\\\alpha\end{bmatrix} = \vec{u} \in \text{pointset of } x+y+z=0$$

when $(1-\alpha) + -\alpha + -\alpha = 0$

so $1 - 3\alpha = 0 \quad \alpha = 1/3$

and $\vec{r} = \begin{bmatrix}1\\0\\0\end{bmatrix} - \begin{bmatrix}2/3\\2/3\\2/3\end{bmatrix} = \begin{bmatrix}1/3\\-2/3\\-2/3\end{bmatrix}$

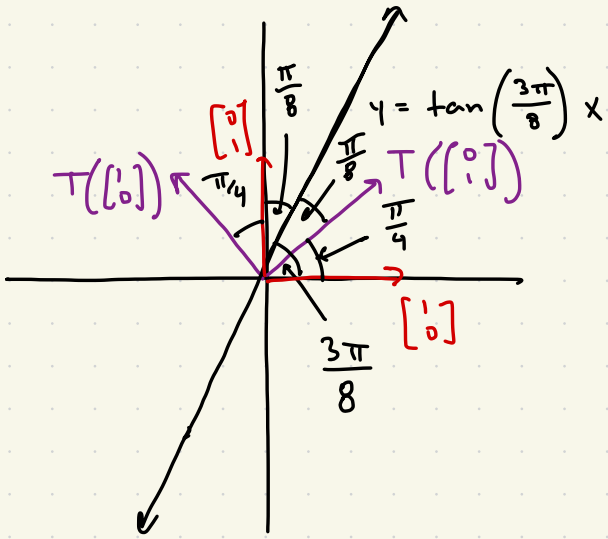
Repeating for each $\vec{e}_1, \vec{e}_2, \vec{e}_3$:

$$\begin{bmatrix} 1/3 & -2/3 & -2/3 \\ -2/3 & 1/3 & -2/3 \\ -2/3 & -2/3 & 1/3 \end{bmatrix}$$

Problem 4.2

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \longmapsto \begin{bmatrix} -\sqrt{2}/2 \\ \sqrt{2}/2 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \longmapsto \begin{bmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \end{bmatrix}$$



$$\begin{bmatrix} -\sqrt{2}/2 & \sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$