

Midterm Solutions

CAs CS 132

Fall 2024

Problem 1 A

$$\begin{bmatrix} 1 & -2 & -7 & -3 \\ 2 & 1 & 6 & 14 \\ 1 & 1 & 5 & 9 \end{bmatrix}$$

Problem 1B

$$\begin{bmatrix} 1 & -2 & -7 & -3 \\ 2 & 1 & 6 & 14 \\ 1 & 1 & 5 & 9 \end{bmatrix}$$

-2 $+4$ $+14$ $+6$
 -1 $+2$ $+7$ $+3$

$$\begin{aligned} R_2 &\leftarrow R_2 - 2R_1 \\ R_3 &\leftarrow R_3 - R_1 \end{aligned}$$

→

$$\begin{bmatrix} 1 & -2 & -7 & -3 \\ 0 & 5 & 20 & 20 \\ 0 & 3 & 12 & 12 \end{bmatrix}$$

$$\begin{aligned} R_2 &\leftarrow R_2 / 5 \\ R_3 &\leftarrow R_3 / 3 \\ R_3 &\leftarrow R_3 - R_2 \end{aligned}$$

→

$$\begin{bmatrix} 1 & -2 & -7 & -3 \\ 0 & 1 & 4 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$+2$ $+8$ $+8$

$$R_1 \leftarrow R_1 + 2R_2$$

→

$$\begin{bmatrix} 1 & 0 & -1 & 5 \\ 0 & 1 & 4 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Problem 1C

$$x_1 = 5 - x_3$$

$$x_2 = 4 - 4x_3$$

x_3 is free

Problem 2

A. False

B. True

C. False

D. False

E. True

F. True

G. True

H. True

I. False

Problem 3

$$\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Problem 4 A

$$\begin{bmatrix} 5 & 7 & 10 \\ 2 & -1 & 4 \\ 0 & -3 & 0 \end{bmatrix}$$

Problem 4B

neither

first and third column are multiples of each other so the columns of A are L.D.

By IMT, A is not one-to-one and not onto

Problem 4C

$$\left\{ \begin{bmatrix} 5 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ -3 \end{bmatrix} \right\}$$

Problem 5A

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \xrightarrow{\substack{+4 \\ -4}} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix} \xrightarrow{\substack{-3 \\ +4 \\ -1}} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 4 & -1 \\ -1 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix} \xrightarrow{\substack{-2 \\ +2}} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 4 & -1 \\ -1 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 6 & -1 \\ -1 & 1 & 0 \\ 3 & -4 & 1 \end{bmatrix}$$

Problem 5 B

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad A^T \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{aligned} a + c &= -2 \\ b + d &= 1 \end{aligned}$$

$$A \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix} \Rightarrow \begin{aligned} b &= -2 \\ d &= 3 \end{aligned}$$

$$\det(A) = 1 \Rightarrow a(3) - (-2)c = 1$$

$$\begin{aligned} a + c &= -2 \\ -3a + 2c &= 1 + 6 \end{aligned} \Rightarrow \begin{aligned} c &= -7 \\ a &= 5 \end{aligned}$$

$$\begin{aligned} a + c &= -2 \\ -c &= 7 \end{aligned}$$

$$A^{-1} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$$