Assignment 1

CS132

Fall 2025

Basic Problems

1. coeff. matrix:

$$\begin{bmatrix} 1 & -2 & -2 \\ 2 & -3 & -5 \\ -2 & 2 & 7 \end{bmatrix}$$

augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & -2 & -2 & 2 \\ 2 & -3 & -5 & 2 \\ -2 & 2 & 7 & -1 \end{array}\right]$$

2.

$$\begin{aligned} x_1 + 2x_2 - x_3 + x_4 &= 7 \\ x_1 + 3x_2 + 2x_4 &= 15 \\ -2x_1 - 6x_2 - 3x_4 &= -27 \end{aligned}$$

3. Plug in values:

$(1) - 2(3) + 2 - 2(3) = 1 - 6 + 2 - 6 = -9$ ✓

$1 - 3 - 2 - 2(3) = 1 - 3 - 2 - 6 = -10$ ✓

$-3(1) + 8(3) - 6(2) + 4(3) = -3 + 24 - 12 + 12 = 21$ ✓

$2(3) - 7(2) + 7(3) = 6 - 14 + 21 = 13$ ✓

4. Augmented matrix:

$$\begin{bmatrix} 1 & -2 & -2 & -7 \\ -1 & 3 & 2 & 10 \\ 2 & -6 & -3 & -18 \end{bmatrix} \xrightarrow{R_2 \leftarrow R_2 + R_1}$$

$$\begin{bmatrix} 1 & -2 & -2 & -7 \\ 0 & 1 & 0 & 3 \\ 2 & -6 & -3 & -18 \end{bmatrix} \xrightarrow{R_3 \leftarrow R_3 - 2R_1}$$

$$\begin{bmatrix} 1 & -2 & -2 & -7 \\ 0 & 1 & 0 & 3 \\ 0 & -2 & 1 & -4 \end{bmatrix} \xrightarrow{R_3 \leftarrow R_3 + 2R_2}$$

$$\begin{bmatrix} 1 & -2 & -2 & -7 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow{\substack{R_1 \leftarrow R_1 + 2R_3 \\ R_1 \leftarrow R_1 + 2R_2}}$$

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$x_1 = 3$

$x_2 = 3$

$x_3 = 2$

$(3, 3, 2)$ is the unique solution.

5.
$$\begin{bmatrix} 9 & 5 & -7 & -5 & -9 \\ 5 & -7 & 1 & -2 & -9 \\ 5 & 1 & -10 & 6 & -5 \\ 5 & 7 & -5 & 2 & 1 \end{bmatrix} \xrightarrow{R_4 \leftarrow -R_4}$$

$$\begin{bmatrix} 9 & 5 & -7 & -5 & -9 \\ 5 & -7 & 1 & -2 & -9 \\ 5 & 1 & -10 & 6 & -5 \\ -5 & -7 & 5 & -2 & -1 \end{bmatrix} \xrightarrow{R_2 \leftarrow R_2 - 2R_3}$$

$$\begin{bmatrix} 9 & 5 & -7 & -5 & -9 \\ 5 & -7 & 1 & -2 & -9 \\ 5 & 1 & -10 & 6 & -5 \\ -5 & -7 & 5 & -2 & -1 \end{bmatrix} \xrightarrow{R_2 \leftarrow R_2 - 5R_4}$$

$$\begin{bmatrix} 9 & 5 & -7 & -5 & -9 \\ -5 & -9 & 21 & -14 & 1 \\ 5 & 1 & -10 & 6 & -5 \\ -5 & -7 & 5 & -2 & -1 \end{bmatrix} \xrightarrow{R_3 \leftarrow R_3 + 3R_4}$$

$$\begin{bmatrix} 9 & 5 & -7 & -5 & -9 \\ 20 & 26 & -4 & -4 & 6 \\ 5 & 1 & -10 & 6 & -5 \\ -5 & -7 & 5 & -2 & -1 \end{bmatrix} \xrightarrow{R_3 \leftrightarrow R_2}$$

$$\begin{bmatrix} 9 & 5 & -7 & -5 & -9 \\ 20 & 26 & -4 & -4 & 6 \\ -10 & -20 & 5 & 0 & -8 \\ -5 & -7 & 5 & -2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 5 & -7 & -5 & -9 \\ -10 & -20 & 5 & 0 & -8 \\ 20 & 26 & -4 & -4 & 6 \\ -5 & -7 & 5 & -2 & -1 \end{bmatrix}$$

6.

$x_1 = -3 - x_2 + 4x_6 - 5x_7$

x_2 is free

$x_3 = -4 - x_6 - 3x_7$

x_4 is free

$x_5 = 2 - 5x_6 + 3x_7$

x_6 is free

x_7 is free

7.

$$\begin{bmatrix} 1 & -1 & -2 & 1 \\ -1 & 2 & 4 & 0 \\ 2 & -3 & -6 & 2 \\ -2 & 1 & 2 & -1 \end{bmatrix}$$

$R_2 \leftarrow R_2 + R_1$

$R_3 \leftarrow R_3 - 2R_1$

$R_4 \leftarrow R_4 + 2R_1$

$$\begin{bmatrix} 1 & -1 & -2 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & -1 & -2 & 0 \\ 0 & -1 & -2 & 1 \end{bmatrix}$$

$R_3 \leftarrow R_3 + R_2$

$R_4 \leftarrow R_4 + R_2$

$$\begin{bmatrix} 1 & -1 & -2 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$R_4 \leftarrow R_4 - 2R_2$

$R_1 \leftarrow R_1 - R_3$

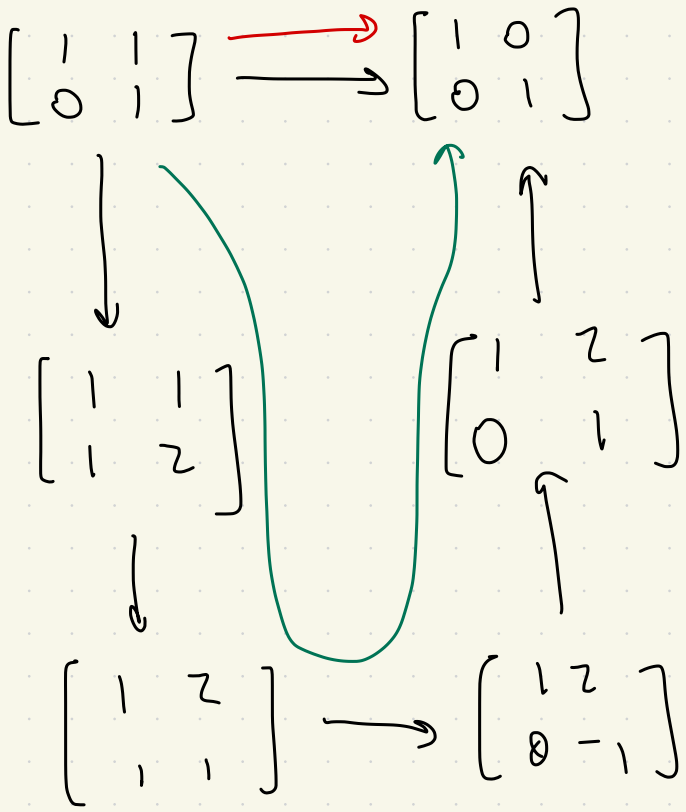
$$\begin{bmatrix} 1 & -1 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \leftarrow R_1 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

True / False

1. True
2. False
3. True
4. True
5. False



6. True

7. False

8. False

$$x+y+z=1$$
$$x+y+z=2$$

9. False

$$x+y=1$$
$$x+y=2$$
$$x+y=3$$

More Difficult Problems

1.

$$\begin{bmatrix} 1 & 4 & -1 \\ 3 & -h & 7 \end{bmatrix} \begin{matrix} \\ \scriptstyle -3 \\ \scriptstyle -12 \\ \scriptstyle +3 \end{matrix} \sim \begin{bmatrix} 1 & 4 & -1 \\ 0 & -(h+12) & 10 \end{bmatrix}$$

The system is inconsistent exactly when $h = -12$.

There are no values of h so that the system has infinitely many solutions. This would require the second equation to be a multiple of the first, which is not possible.

2. $\begin{bmatrix} h & 2 & 1 \\ 3 & 9 & k \end{bmatrix}$

a) $h = 2/3$

$k = 0$

$$\begin{bmatrix} 2/3 & 2 & 1 \\ 3 & 9 & 0 \end{bmatrix} \sim$$

$$\begin{bmatrix} 2 & 6 & 1 \\ 3 & 9 & 0 \end{bmatrix} \sim$$

$$\begin{bmatrix} 6 & 18 & 3 \\ 6 & 18 & 0 \end{bmatrix} \sim \begin{bmatrix} 2 & 6 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

b) $h = 0$

$k = 0$

$$\begin{bmatrix} 0 & 2 & 1 \\ 3 & 9 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1/2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -3/2 \\ 0 & 1 & 1/2 \end{bmatrix}$$

c) $h = 2/3$

$k = 9/2$

$$\begin{bmatrix} 2/3 & 2 & 1 \\ 3 & 9 & 9/2 \end{bmatrix} \sim \begin{bmatrix} 6 & 18 & 9 \\ 6 & 18 & 9 \end{bmatrix}$$

$$\sim \begin{bmatrix} 6 & 18 & 9 \\ 0 & 0 & 0 \end{bmatrix}$$