Assignment 5

CS132

Fall 2025

Basic Problems

(1)

$$\begin{bmatrix} 1 & 1 & 2 \\ \overset{-2}{2} & \overset{-2}{3} & \overset{-4}{5} \\ \overset{+1}{-1} & \overset{+1}{-3} & \overset{+2}{-3} \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & \overset{+2}{-2} & \overset{+2}{-1} \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

both one-to-one and onto

(2) A has more rows than columns, so it cannot be onto

The columns of A are not colinear so they form a linearly independant set.

one-to-one and not onto

(3)

$$\begin{bmatrix} 1 & 2 & -1 & 6 & 9 \\ \overset{-2}{2} & \overset{-4}{5} & \overset{+2}{-5} & \overset{-12}{13} & \overset{-18}{22} \\ \overset{+3}{-3} & \overset{+6}{-4} & \overset{-3}{-3} & \overset{+18}{-16} & \overset{+27}{-19} \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 & 6 & 9 \\ 0 & 1 & -3 & 1 & 4 \\ 0 & 2 & -6 & 2 & 8 \end{bmatrix} \sim$$

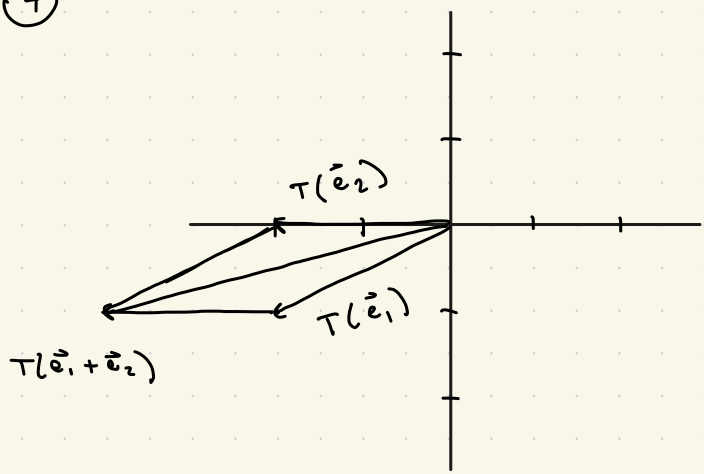
$$\begin{bmatrix} 1 & 2 & -1 & 6 & 9 \\ 0 & 1 & -3 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

A has more columns than rows so the transformation cannot be one-to-one

It has fewer pivots than rows

neither one-to-one nor onto

(4)



5

$$2\begin{bmatrix}6 & 1\\ -1 & -3\\ 8 & 1\end{bmatrix} - 4\begin{bmatrix}6 & -5\\ -6 & -2\\ -3 & 4\end{bmatrix} - 3\begin{bmatrix}-4 & 0\\ -8 & 5\\ -4 & -1\end{bmatrix} =$$

$$\begin{bmatrix}12 & 2\\ -2 & -6\\ 16 & 2\end{bmatrix} - \begin{bmatrix}24 & -20\\ -24 & -8\\ -12 & 16\end{bmatrix} - \begin{bmatrix}-12 & 0\\ -24 & 15\\ -12 & -3\end{bmatrix} =$$

$$\begin{bmatrix}(12-24+12) & 2+20\\ (-2+24+24) & (-6+8-15)\\ (16+12+12) & (2-16+3)\end{bmatrix} = \begin{bmatrix}0 & 22\\ 46 & -13\\ 40 & -11\end{bmatrix}$$

6

$$\begin{bmatrix}-2 & 6\\ -2 & -7\\ -3 & -2\end{bmatrix}\begin{bmatrix}-4\\ -10\end{bmatrix} = -4\begin{bmatrix}-2\\ -2\\ -3\end{bmatrix} - 10\begin{bmatrix}6\\ -7\\ -2\end{bmatrix}$$

$$= \begin{bmatrix}8\\ 8\\ 12\end{bmatrix} + \begin{bmatrix}-60\\ 70\\ 20\end{bmatrix} = \begin{bmatrix}-52\\ 78\\ 32\end{bmatrix}$$

$$\begin{bmatrix} -2 & 6 \\ -2 & -7 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} 8 \\ -4 \end{bmatrix} = 8 \begin{bmatrix} -2 \\ -2 \\ -3 \end{bmatrix} - 4 \begin{bmatrix} 6 \\ -7 \\ -2 \end{bmatrix} = \begin{bmatrix} -16 \\ -16 \\ -24 \end{bmatrix} + \begin{bmatrix} -24 \\ 28 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} -40 \\ 12 \\ -16 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 6 \\ -2 & -7 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} 4 \\ 9 \end{bmatrix} = 4 \begin{bmatrix} -2 \\ -2 \\ -3 \end{bmatrix} + 9 \begin{bmatrix} 6 \\ -7 \\ -2 \end{bmatrix} = \begin{bmatrix} -8 \\ -8 \\ -12 \end{bmatrix} + \begin{bmatrix} 54 \\ -63 \\ -18 \end{bmatrix}$$

$$= \begin{bmatrix} 46 \\ -71 \\ -30 \end{bmatrix}$$

$$\begin{bmatrix} -52 & -40 & 46 \\ 78 & 12 & -71 \\ 32 & -16 & -30 \end{bmatrix}$$

7

$$\begin{bmatrix} 8 & 6 & -1 & -2 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 1 \\ 5 \end{bmatrix} = 24 + 30 - 1 - 10$$

$$= 43$$

8

$$\begin{bmatrix} -3 \\ 6 \\ 8 \\ 5 \end{bmatrix} \begin{bmatrix} -9 & -7 & 0 & -8 \end{bmatrix} = \begin{bmatrix} 27 & 21 & 0 & 24 \\ -54 & -42 & 0 & -48 \\ -72 & -56 & 0 & -64 \\ -45 & -35 & 0 & -40 \end{bmatrix}$$

9 This matrix multiplication is not well-defined. A has 2 columns and B has 4 rows. These counts don't match

10 $A\vec{x} = \vec{b}_1$

$$\vec{x} = \begin{bmatrix} -10 & -10 & -1 \\ -2 & -4 & -3 \\ -8 & -6 & 1 \end{bmatrix} \begin{bmatrix} -5 \\ 6 \\ -1 \end{bmatrix}$$

$$= -5 \begin{bmatrix} -10 \\ -2 \\ -8 \end{bmatrix} + 6 \begin{bmatrix} -10 \\ -4 \\ -6 \end{bmatrix} - \begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 50 \\ 10 \\ 40 \end{bmatrix} + \begin{bmatrix} -60 \\ -24 \\ -36 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -9 \\ -11 \\ 3 \end{bmatrix}$$

$$A\vec{x} = \vec{b}_2$$

$$\vec{x} = \begin{bmatrix} -10 & -10 & -1 \\ -2 & -4 & -3 \\ -8 & -6 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -3 \\ -10 \end{bmatrix}$$

$$= \begin{bmatrix} 10 \\ 2 \\ 8 \end{bmatrix} + 3 \begin{bmatrix} 10 \\ 4 \\ 6 \end{bmatrix} + 10 \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} =$$

$$= \begin{bmatrix} 10 + 30 + 10 \\ 2 + 12 + 30 \\ 8 + 18 - 10 \end{bmatrix} = \begin{bmatrix} 50 \\ 44 \\ 16 \end{bmatrix}$$

$$A\vec{x} = \vec{b}_3$$

$$\vec{x} = \begin{bmatrix} -10 & -10 & -1 \\ -2 & -4 & -3 \\ -8 & -6 & 1 \end{bmatrix} \begin{bmatrix} -6 \\ 4 \\ -4 \end{bmatrix} =$$

$$6 \begin{bmatrix} 10 \\ 2 \\ 8 \end{bmatrix} + 4 \begin{bmatrix} -10 \\ -4 \\ -6 \end{bmatrix} + 4 \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} =$$

$$\begin{bmatrix} 60 - 40 + 4 \\ 12 - 16 + 12 \\ 48 - 24 - 4 \end{bmatrix} = \begin{bmatrix} 24 \\ 8 \\ 20 \end{bmatrix}$$

(11)

$$\begin{bmatrix} -8 & -2 \\ -2 & -3 \end{bmatrix}^{-1} = \frac{1}{24-4}\begin{bmatrix} -3 & 2 \\ 2 & -8 \end{bmatrix} = \boxed{\begin{bmatrix} -3/20 & 1/10 \\ 1/10 & -2/5 \end{bmatrix}}$$

$$\left[\begin{array}{cc|cc} -8 & -2 & 1 & 0 \\ -2 & -3 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 2 & 3 & 0 & -1 \\ \overset{-8}{8} & \overset{-12}{2} & -1 & \overset{+4}{0} \end{array}\right] \sim$$

$$\left[\begin{array}{cc|cc} 2 & 3 & 0 & -1 \\ 0 & -10 & -1 & 4 \end{array}\right] \sim \left[\begin{array}{cc|cc} 2 & \overset{-3}{3} & \overset{-3/10}{0} & \overset{+6/5}{-1} \\ 0 & 1 & 1/10 & -2/5 \end{array}\right] \sim$$

$$\left[\begin{array}{cc|cc} 2 & 0 & -3/10 & 1/5 \\ 0 & 1 & 1/10 & -2/5 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 0 & -3/20 & 1/10 \\ 0 & 1 & 1/10 & -2/5 \end{array}\right]$$

True / False

1. True
2. True
3. False $\quad \vec{x} \mapsto \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$
4. True
5. False $\quad A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

6 False $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

7 True

8 True

$\boxed{\text{More Difficult Problems}}$

1

$$\begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 4 \begin{bmatrix} 2 \\ -3 \end{bmatrix} + 3 \begin{bmatrix} 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ -12 \end{bmatrix} + \begin{bmatrix} 15 \\ 3 \end{bmatrix} =$$

$$\begin{bmatrix} 23 \\ -9 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} -5 \\ k \end{bmatrix} = -5 \begin{bmatrix} 2 \\ -3 \end{bmatrix} + k \begin{bmatrix} 5 \\ 1 \end{bmatrix} = \begin{bmatrix} -10 + 5k \\ 15 + k \end{bmatrix}$$

$$AB = \begin{bmatrix} 23 & -10 + 5k \\ -9 & 15 + k \end{bmatrix}$$

$$\begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix} \begin{bmatrix} 2 \\ -3 \end{bmatrix} = 2 \begin{bmatrix} 4 \\ 3 \end{bmatrix} + 3 \begin{bmatrix} 5 \\ -k \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix} + \begin{bmatrix} 15 \\ -3k \end{bmatrix} = \begin{bmatrix} 23 \\ 6-3k \end{bmatrix}$$

$$\begin{bmatrix} 4 & -5 \\ 3 & K \end{bmatrix} \begin{bmatrix} 5 \\ 1 \end{bmatrix} = 5 \begin{bmatrix} 4 \\ 3 \end{bmatrix} + \begin{bmatrix} -5 \\ K \end{bmatrix} = \begin{bmatrix} 20 \\ 15 \end{bmatrix} + \begin{bmatrix} -5 \\ K \end{bmatrix}$$

$$= \begin{bmatrix} 15 \\ 15+K \end{bmatrix}$$

$$BA = \begin{bmatrix} 23 & 15 \\ 6-3K & 15+K \end{bmatrix}$$

$AB = BA$ when

$$\begin{bmatrix} 23 & -10+5K \\ -9 & 15+K \end{bmatrix} = \begin{bmatrix} 23 & 15 \\ 6-3K & 15+K \end{bmatrix}$$

$$\left.\begin{aligned} 5K - 10 &= 15 \\ -3K + 6 &= -9 \end{aligned}\right\} \; K = 5$$

(2) $A^2 = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$A^6 = A^3 A^3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^{-1} = A^5$

(3) $A^{18} = I$ because this represents rotation around the z-axis by 2π.

$A^{-1} = A^{17}$