Quiz 2 (Version 2)

`CAS CS 132`: *Geometric Algorithms*

September 29, 2025

Name:

BUID:

- ▷ You will have approximately 30 minutes to complete this exam.
- ▷ Your final solution must appear in the solution boxes for each problem. **Only include your final solution in the solution box. You must show your work outside of the solution box.** You will not recieve credit it you don't show your work.

1 Linear Dependence

Determine if the following vectors are linearly dependent. If they are write a dependence relation, i.e., determine a linear combination of the given vectors which sums to **0**.

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 8 \\ -3 \\ -15 \end{bmatrix} \quad \mathbf{v}_3 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 8 & -1 \\ 0 & -3 & 1 \\ -3^{\,+3} & -15^{\,+24} & 0^{\,-3} \end{bmatrix} \sim \begin{bmatrix} 1 & 8 & -1 \\ 0 & -3 & 1 \\ 0 & 9 & -3 \end{bmatrix}$$

$$\begin{array}{ccc} & -8 & +8/3 \end{array}$$

$$\sim \begin{bmatrix} 1 & 8 & -1 \\ 0 & 1 & -1/3 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 5/3 \\ 0 & 1 & -1/3 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = -5$

$x_2 = 3$

$x_3 = 1$

Solution.

$$-5\vec{v}_1 + 3\vec{v}_2 + \vec{v}_3 = \vec{0}$$

2 Matrix Equations

Determine a general form solution for the matrix equation $A\mathbf{x} = \mathbf{b}$ where A and $\mathbf{b}$ are given below. If the equation has no solutions then write *NO SOLUTION*.

$$A = \begin{bmatrix} 1 & 4 & -2 \\ -1 & -4 & 3 \\ -3 & -12 & 7 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 3 \\ -7 \\ -13 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & -2 & 3 \\ -1^{+1} & -4^{+4} & 3^{-2} & -7^{+3} \\ -3^{+3} & -12^{+12} & 7^{-6} & -13^{+9} \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & -2 & 3 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 1 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & -2^{+2} & 3^{-8} \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 0 & -5 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Solution.

$x_1 = -5 - 4x_2$

x_2 is free

$x_3 = -4$

3 Linear Equations and Spans

Determine a linear equation whose point set is the span of the following vectors. The linear equation you determine should have relatively prime integer coefficients (i.e., it should not be possible to divide the equation by an integer value and get a new equation with integer coefficients).

$$\mathbf{v}_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & b_1 \\ 1 & -1 & b_2 \\ 1 & 1 & b_3 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & b_2 \\ 0 & 1 & b_1 \\ 1^{-1} & 1^{+1} & b_3 - b_2 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & b_2 \\ 0 & 1 & b_1 \\ 0 & 2^{-2} & b_3 - b_2 \; {}^{-2b_1} \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & b_2 \\ 0 & 1 & b_1 \\ 0 & 0 & b_3 - b_2 - 2b_1 \end{bmatrix}$$

Solution.

$$-2x_1 - x_2 + x_3 = 0$$