Quiz 3 (Version 3)

`CAS CS 132`: *Geometric Algorithms*

October 14, 2025

Name:

BUID:

- ▷ You will have approximately 30 minutes to complete this exam.
- ▷ Your final solution must appear in the solution boxes for each problem. **Only include your final solution in the solution box. You must show your work outside of the solution box.** You will not recieve credit it you don't show your work.

1 Matrix for a Linear Transformation

Let T be a linear transformation with the following input-output behavior.

$$T\left(\begin{bmatrix}-2\\-2\end{bmatrix}\right)=\begin{bmatrix}-6\\2\end{bmatrix}\quad T\left(\begin{bmatrix}-1\\-2\end{bmatrix}\right)=\begin{bmatrix}-4\\3\end{bmatrix}$$

Determine the matrix that implements T, i.e., determine the matrix A such that $T(\mathbf{v}) = A\mathbf{v}$ for all vectors $\mathbf{v}$ in the domain of T.

$$\begin{bmatrix}-2 & -1 & 1\\ \overset{+2}{-2} & \overset{+1}{-2} & \overset{-1}{0}\end{bmatrix} \sim \begin{bmatrix}-2 & \overset{+1}{-1} & \overset{+1}{1}\\ 0 & -1 & -1\end{bmatrix} \sim \begin{bmatrix}-2 & 0 & 2\\ 0 & 1 & 1\end{bmatrix} \sim \begin{bmatrix}1 & 0 & -1\\ 0 & 1 & 1\end{bmatrix}$$

$$\begin{bmatrix}-2 & -1 & 0\\ \overset{+2}{-2} & \overset{+1}{-2} & 1\end{bmatrix} \sim \begin{bmatrix}-2 & -1 & 0\\ 0 & -1 & 1\end{bmatrix} \sim \begin{bmatrix}-2 & 0 & -1\\ 0 & 1 & -1\end{bmatrix} \sim \begin{bmatrix}1 & 0 & \frac{1}{2}\\ 0 & 1 & -1\end{bmatrix}$$

$$-\begin{bmatrix}-6\\2\end{bmatrix}+\begin{bmatrix}-4\\3\end{bmatrix}=\begin{bmatrix}6\\-2\end{bmatrix}+\begin{bmatrix}-4\\3\end{bmatrix}=\begin{bmatrix}2\\1\end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix}-6\\2\end{bmatrix}-\begin{bmatrix}-4\\3\end{bmatrix}=\begin{bmatrix}-3\\1\end{bmatrix}+\begin{bmatrix}4\\-3\end{bmatrix}=\begin{bmatrix}1\\-2\end{bmatrix}$$

Solution.

$$\begin{bmatrix}2 & 1\\ 1 & -2\end{bmatrix}$$

2 One-to-one and Onto

Determine if the following linear transformation is one-to-one, onto, both, or neither. You must show your work.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mapsto \begin{bmatrix} x_1 - 2x_3 \\ -3x_1 + x_2 + 4x_3 \\ -x_1 - 3x_2 + 9x_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -3 \\ -3^{+3} & 1^{+0} & 4^{-9} \\ -1^{+1} & -3^{+0} & 9^{-3} \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -5 \\ 0 & 1^{-1} & -2^{+5} \end{bmatrix} \sim \begin{bmatrix} \boxed{1} & 0 & -3 \\ 0 & \boxed{1} & -5 \\ 0 & 0 & \boxed{3} \end{bmatrix}$$

Solution.

both. one pivot per row and column

3 Using inverses

Let A be matrix such that A^{-1} is defined as below. Use this to determine the solution to the matrix equations of the form $A\mathbf{x} = \mathbf{b}_i$, where each $\mathbf{b}_i$ is defined below.

$$A^{-1} = \begin{bmatrix} -3 & 1 & -2 \\ -2 & 1 & -1 \\ -2 & -3 & 2 \end{bmatrix} \quad \mathbf{b}_1 = \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix} \quad \mathbf{b}_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$A^{-1}\vec{b}_1 = -2\begin{bmatrix} -3 \\ -2 \\ -2 \end{bmatrix} + 2\begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 4 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \\ -6 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \\ -2 \end{bmatrix}$$

$$A^{-1}\vec{b}_2 = -1\begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 3 \end{bmatrix}$$

Solution.

$$x_1 = \begin{bmatrix} 8 \\ 6 \\ -2 \end{bmatrix} \qquad x_2 = \begin{bmatrix} -1 \\ -1 \\ 3 \end{bmatrix}$$