Quiz 4 (Version 4)

CAS CS 132: *Geometric Algorithms*

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- ▷ You will have approximately 30 minutes to complete this exam.
- ▷ Your final solution must appear in the solution boxes for each problem. **Only include your final solution in the solution box. You must show your work outside of the solution box.** You will not recieve credit it you don't show your work.

1 LU Factorization

Determine the LU factorization of the following matrix using the process that we discussed in class.

$$\begin{bmatrix} 1 & -2 & 9 \\ 0 & 1 & -5 \\ -1 & 4 & -19 \end{bmatrix}$$

$R_3 \leftarrow R_3 + R_1$

$$\begin{bmatrix} 1 & -2 & 9 \\ 0 & 1 & -5 \\ 0 & 2 & -10 \end{bmatrix}$$

$R_3 \leftarrow R_3 - 2R_2$

$$\begin{bmatrix} 1 & -2 & 9 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution.

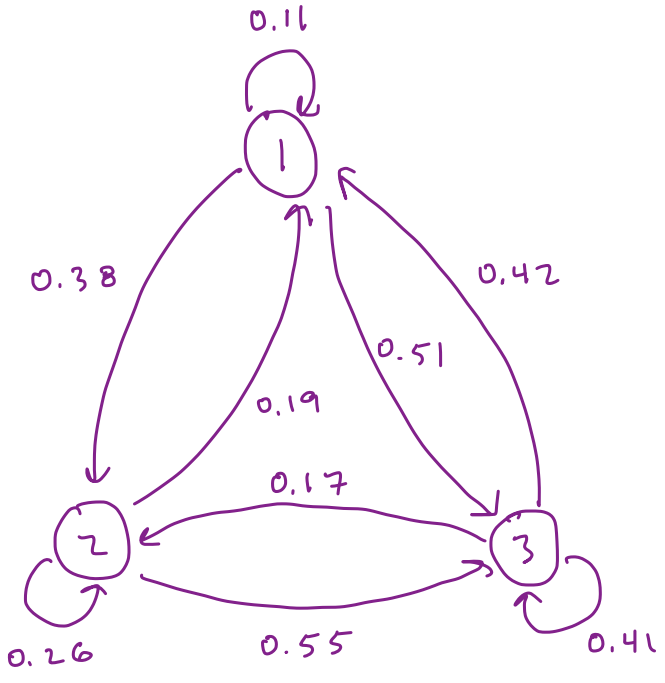
$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 2 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & -2 & 9 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

2 State Diagrams

Draw the state diagram with the following transition matrix. You should label each state in your diagram with the index of its corresponding column. That is, the state corresponding to the first column should be labeled "1" and the state corresponding to the second column should be labeled "2" and so on.

$$\begin{bmatrix} 0.11 & 0.19 & 0.42 \\ 0.38 & 0.26 & 0.17 \\ 0.51 & 0.55 & 0.41 \end{bmatrix}$$

Solution.



3 Steady-State Distributions

Determine $\lim_{k\to\infty} A^k \begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix}$ where A is defined as follows.

$$A = \begin{bmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{bmatrix}$$

$A - I = \begin{bmatrix} 2/10 & 8/10 \\ 8/10 & 2/10 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -8/10 & 8/10 \\ 8/10 & -8/10 \end{bmatrix} \sim \begin{bmatrix} 8 & -8 \\ 8 & -8 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$ $x_1 = x_2$, x_2 is free

$x_1 + x_2 = 1 \Rightarrow 2x_2 = 1 \Rightarrow x_1 = 1/2 \quad x_2 = 1/2$

Solution.

$$\begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$