Quiz 5 (Version 5)

`CAS CS 132`: *Geometric Algorithms*

November 17, 2025

Name: Nathan Mull

BUID: 12345678

- ▷ You will have approximately 30 minutes to complete this exam.
- ▷ Your final solution must appear in the solution boxes for each problem. **Only include your final solution in the solution box. You must show your work outside of the solution box.** You will not recieve credit it you don't show your work.

1 Column Space and Null Space

Determine bases for the column space and the null space of the following matrix. Note that you are given its RREF.

$$\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{a}_4 & \mathbf{a}_5 & \mathbf{a}_6 & \mathbf{a}_7 \end{bmatrix} \sim \begin{bmatrix} 1 & 5 & -4 & 0 & 0 & 6 & -1 \\ 0 & 0 & 0 & 1 & 0 & 5 & -5 \\ 0 & 0 & 0 & 0 & 1 & -2 & -5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 = -5x_2 + 4x_3 - 6x_6 + x_7$

x_2 is free

x_3 is free

$x_4 = -5x_6 + 5x_7$

$x_5 = 2x_6 + 5x_7$

x_6 is free

x_7 is free

Solution.

$\text{Col}(A) = \text{span}\{\vec{a}_1, \vec{a}_4, \vec{a}_5\}$

$$\text{Nul}(A) = \text{span}\left\{ \begin{bmatrix} -5 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 0 \\ 0 \\ -5 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 5 \\ 5 \\ 0 \\ 1 \end{bmatrix} \right\}$$

2 Coordinate Vectors

Determine the coordinate vector $[\mathbf{u}]_{\mathcal{B}}$ where $\mathbf{u}$ and $\mathcal{B}$ are defined below.

$$\mathbf{u} = \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix} \qquad \mathcal{B} = \left\{ \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 9 \\ 3 \\ 6 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 3 & 9 & 3 \\ 2 & 3 & -1 \\ 3 & 6 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 1 \\ 2 & 3 & -1 \\ 1 & 2 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 1 \\ 0 & -3 & -3 \\ 0 & -1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution.

$$[\vec{u}]_B = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

3 Eigenbases

For the following matrix, determine all eigenvalues and bases for the corresponding eigenspaces.

$$\begin{bmatrix} -6 & -5 \\ 10 & 9 \end{bmatrix}$$

$(\lambda+6)(\lambda-9)+50 = \lambda^2-3\lambda-54+50 = \lambda^2-3\lambda-4 = (\lambda-4)(\lambda+1)$

$$\begin{bmatrix} -10 & -5 \\ 10 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 1/2 \\ 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} -5 & -5 \\ 10 & 10 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Solution.

$\lambda_1 = 4 \qquad \left\{ \begin{bmatrix} -1 \\ 2 \end{bmatrix} \right\}$

$\lambda_2 = 1 \qquad \left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$