Quiz 6 (Version 6)

`CAS CS 132`: *Geometric Algorithms*

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- ▷ You will have approximately 30 minutes to complete this exam.
- ▷ Your final solution must appear in the solution boxes for each problem. **Only include your final solution in the solution box. You must show your work outside of the solution box.** You will not recieve credit it you don't show your work.

1 Change of Basis Matrix

Determine the change-of-basis matrix for the following basis.

$$\left\{ \begin{bmatrix} -1 \\ -3 \end{bmatrix}, \begin{bmatrix} -3 \\ -8 \end{bmatrix} \right\}$$

$$\begin{bmatrix} -1 & -3 \\ -3 & -8 \end{bmatrix}^{-1} = \frac{1}{8-9} \begin{bmatrix} -8 & 3 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 8 & -3 \\ -3 & 1 \end{bmatrix}$$

Solution.

$$\begin{bmatrix} 8 & -3 \\ -3 & 1 \end{bmatrix}$$

2 Diagonalization

Determine a diagonalization of the following matrix, if it exists. You can leave the rightmost factor in the form P^{-1}, i.e., you don't have to compute the inverse of P.

$$\begin{bmatrix} 8 & -6 \\ 9 & -7 \end{bmatrix}$$

$(8-\lambda)(-7-\lambda)+54 = -56-8\lambda+7\lambda+\lambda^2+54 = \lambda^2-\lambda-2 = (\lambda-2)(\lambda+1)$

$\lambda = 2, -1$

$A - 2I = \begin{bmatrix} 6 & -6 \\ 9 & -9 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$A + I = \begin{bmatrix} 9 & -6 \\ 9 & -6 \end{bmatrix} \sim \begin{bmatrix} 1 & -2/3 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$P = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \quad D = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \quad P^{-1} = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$

Solution.

$$\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$$

3 Unit Vectors

Determine the unit-length normalizations of **v**. You must simplify the expression as much as you can.

$$\mathbf{v} = \begin{bmatrix} 0 \\ -3 \\ 2 \\ 3 \end{bmatrix}$$

$\|v\| = \sqrt{9 + 4 + 9} = \sqrt{23}$

Solution.

$$\frac{1}{\sqrt{23}} \begin{bmatrix} 0 \\ -3 \\ 2 \\ 3 \end{bmatrix}$$