Problem 1

$$A = \begin{array}{c} \begin{bmatrix} 0 & 2 & 0 & 4 & 1 \\ 0 & -3 & 1 & -3 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 3 & 0 \end{bmatrix} \\ \begin{array}{ccccc} \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ \vec{a}_1 & \vec{a}_2 & \vec{a}_3 & \vec{a}_4 & \vec{a}_5 \end{array} \end{array}$$

(notation for columns)

(A) Write down a basis for Col A.

(B) Write down a basis for Nul A.

(C) Write down a nontrivial linear dependence relation between the columns of A.

(D) If A is interpreted as the augmented matrix of an (nonhomogeneous) linear system, how many solutions does the system have?

(E) Does this system have a least squares solution? If so, is it unique?

Problem 2

A is a 7×5 matrix.

$$A^T A = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(A) What are the singular values of A?

(B) What is nullity $A = \dim \operatorname{Nul} A$? Use the fact proven in lecture that $\operatorname{Nul} A = \operatorname{Nul} A^T A$.

(C) What is $\operatorname{rk} A = \dim \operatorname{Col} A$?

(D) What is $\operatorname{rk} A^T$ & nullity A^T? (Hint: row operations preserve Row A, the row space of A)

(E) Write down a V^T in the SVD of A, assuming ordering of singular values from greatest to least.

Problem 3

$$\mathcal{B} = \left\{ \underbrace{\begin{bmatrix} 3 \\ 7 \end{bmatrix}}_{=\vec{b}_1}, \underbrace{\begin{bmatrix} -2 \\ -4 \end{bmatrix}}_{=\vec{b}_2} \right\}$$

(A) Write down a matrix that implements the change of basis from the standard basis to $\mathcal{B}$.

(B) Construct an orthonormal basis $\mathcal{C}$ that contains a scalar multiple of $\vec{b}_2$.

(C) Write down the 2×2 matrix (in std basis) that implements the linear transformation that maps:

$$\vec{c}_1 \mapsto 3\vec{c}_1$$
$$\vec{c}_2 \mapsto -\vec{c}_2$$