# More Practice Problems

CAS CS 132: Geometric Algorithms

## Intersection of Spans

Determine a vector with integer entries which appears in *both* of the following spans.

$$\mathsf{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\} \qquad \mathsf{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \right\}$$

## Dependence Relations

Suppose that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly independent set of vectors and that

$$\begin{aligned} \mathbf{u}_1 &= \mathbf{v}_1 + \mathbf{v}_3 \\ \mathbf{u}_2 &= -2\mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3 \\ \mathbf{u}_3 &= -3\mathbf{v}_1 - \mathbf{v}_2 - 6\mathbf{v}_3 \end{aligned}$$

Determine a dependence relation with integer weights for the vectors $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$.

## True/False

Determine if each of the following statements are true or false. If it is false, give a counterexample.

1. For any matrices $A$ and $B$, if $AB = I$ then $A$ is invertible and $B = A^{-1}$.
2. For any matrix $A$ in $\mathbb{R}^{10 \times 15}$ and any $\mathbf{b}$ in $\mathbb{R}^{10}$, the matrix equation $A\mathbf{x} = \mathbf{b}$ has a solution.

3. For any matrices $A$ and $B$, if $AB = 0$, then $A = 0$ or $B = 0$.

4. For any vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, if $\mathbf{v}_1 \in \mathsf{span}\{\mathbf{v}_2, \mathbf{v}_3\}$ then $\{\mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_1 + \mathbf{v}_3\}$ is a linearly dependent set.

5. For any matrices $A$ and $B$, if $AB = BA$, then $A = B$.

## Inverses(?)

### Multiply on the Right

Determine a matrix $B$ with integer entries such that the following equality holds.

$$\begin{bmatrix} 1 & -1 & 2 \\ -3 & 4 & 2 \end{bmatrix} B = I$$

### Multiply on the Left

Explain why it is not possible to determine a matrix $B$ such that the following equality holds.

$$B \begin{bmatrix} 1 & -1 & 2 \\ -3 & 4 & 2 \end{bmatrix} = I$$

## Linear Transformations

Suppose that $T : \mathbb{R}^3 \to \mathbb{R}^3$ is the linear transformation which reflects vectors across the $xy$ plane (i.e., across the plane given by the linear equation $z = 0$) and that $S : \mathbb{R}^3 \to \mathbb{R}^3$ the transformation which rotates vectors around $\mathsf{span}\{[1 \quad 1 \quad 0]^T\}$ by 180 degrees. Determine the matrix which implements $S \circ T$, the composition of $S$ and $T$ (recall that $(S \circ T)(\mathbf{v}) = S(T(\mathbf{v}))$).