Practice Midterm Exam

CAS CS 132: Geometric Algorithms

Name:

BUID:

- ▷ You will have approximately 75 minutes to complete this exam. Make sure to read every question, some are easier than others.
- ▷ Do not remove any pages from the exam.
- ▷ Your final solution must appear in the solution boxes for each problem. **Only include your final solution in the solution box. You must show your work outside of the solution box.** You will not recieve credit it you don't show your work.
- ▷ We will not look at any work on the pages marked "*This page is intentionally left blank.*" You should use these pages for scratch work.

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1 Linear Equations and Spans

(6 points) Determine a linear equation whose solution set is the span of the following two vectors. (*Note.* You may not use the cross product for this problem.)

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \qquad \mathbf{v}_2 = \begin{bmatrix} 3 \\ -2 \\ 3 \end{bmatrix}$$

Solution.

2 Pseudoinverses

(6 points) Determine a matrix A and two values k and h such that

$$A\begin{bmatrix} 1 & -1 \\ 3 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \text{and} \qquad \begin{bmatrix} 1 & -1 \\ 3 & -2 \\ -1 & 3 \end{bmatrix} A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ h & k & 0 \end{bmatrix}$$

(*Hint.* First determine the rightmost column of A.)

Solution.

$A =$ $\qquad$ $h =$ $\qquad$ $k =$

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3 Linear Transformations

Consider the following linear transformations.

$$S\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 \\ -x_2 \\ x_1 + x_2 \end{bmatrix} \qquad T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 + x_3 \\ -x_1 - 2x_2 - 3x_3 \end{bmatrix}$$

A. (3 points) Determine the matrix that implements $S \circ T$, the composition of S and T. That is, determine the matrix A such that the transformation $\mathbf{x} \mapsto S(T(\mathbf{x}))$ is equivalent to the transformation $\mathbf{x} \mapsto A\mathbf{x}$.

Solution.

B. (6 points) Determine a general form solution that describes the preimages of the following vector $\mathbf{b}$ under the linear transformation $S \circ T$. That is determine a general form solution where $S(T(\mathbf{x})) = \mathbf{b}$ if and only if $\mathbf{x}$ is in the solution set described by your general form solution.

$$\mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$$

Solution.

x_1

x_2

x_3

4 True/False

Determine if each of the following statements is **True** or **False**. Bubble in your answers below. You do not need to show your work.

A. (1 point) A system of linear equations with 3 variables and 4 equations cannot be consistent.

○ True
○ False

B. (1 point) For any matrix $A \in \mathbb{R}^{n \times n}$, if $A = LU$ where L and U form an LU-factorization of A, and if L is invertible, then so is A.

○ True
○ False

C. (1 point) For any matrices $A, B \in \mathbb{R}^{n \times n}$, if A and B are invertible, then so is AB.

○ True
○ False

D. (1 point) If A is the augmented matrix of a linear system and it has a pivot position in every column, then the system is inconsistent.

○ True
○ False

E. (1 point) If the equation $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution, then it has infinitely many solutions.

○ True
○ False

F. (1 point) For any vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \in \mathbb{R}^n$, if $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent, then so is $\{\mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_3\}$.

○ True
○ False

G. (1 point) For any matrix $A \in \mathbb{R}^{m \times n}$, the matrix AA^T is symmetric (recall that a matrix B is symmetric if $B = B^T$).

○ True
○ False

H. (1 point) For any matrix $A \in \mathbb{R}^{m \times n}$, if $m < n$, and the columns of A span $\mathbb{R}^m$, then the reduced echelon form of A has only 0s and 1s.

○ True
○ False

I. (1 point) For any matrices $A, B \in \mathbb{R}^{n \times n}$, if $AB = I$, then $BA = I$.

○ True
○ False

5 Linear Dependence

(6 points) Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ be vectors such that

$$\begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{v}_4 \end{bmatrix} \sim \begin{bmatrix} 2 & -3 & 3 & 8 \\ 0 & -2 & -2 & 7 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Determine a dependence relation for the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$. That is, determine coefficients $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ such that $\alpha_1\mathbf{v}_1 + \alpha_2\mathbf{v}_2 + \alpha_3\mathbf{v}_3 + \alpha_4\mathbf{v}_4 = \mathbf{0}$.

Solution.

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